

$$\int f'(x) [f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1} + C$$

$$\int f'(x) [f(x)]^{\frac{1}{n}} dx = \int f'(x) \sqrt[n]{f(x)} dx = \frac{[f(x)]^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \frac{f'(x)}{f(x) \cdot \ln a} dx = \log_a |f(x)| + C; \quad a > 0, a \neq 1$$

$$\int e^{f(x)} \cdot f'(x) dx = e^{f(x)} + C$$

$$\int a^{f(x)} \cdot f'(x) \cdot \ln a dx = a^{f(x)} + C \quad a > 0, a \neq 1$$

$$\int \cos[f(x)] \cdot f'(x) dx = \text{sen}[f(x)] + C$$

$$\int \text{sen}[f(x)] \cdot f'(x) dx = -\cos[f(x)] + C$$

$$\int \text{tg}[f(x)] \cdot f'(x) dx = -\ln[\cos f(x)] + C$$

$$\int \text{cot g}[f(x)] \cdot f'(x) dx = \ln[\text{sen } f(x)] + C$$

$$\int \text{ch}[f(x)] \cdot f'(x) dx = \text{sh}[f(x)] + C$$

$$\int \text{sh}[f(x)] \cdot f'(x) dx = \text{ch}[f(x)] + C$$

$$\int \text{th}[f(x)] \cdot f'(x) dx = \ln[\text{ch } f(x)] + C$$

$$\int \text{coth}[f(x)] \cdot f'(x) dx = \ln[\text{sh } f(x)] + C$$

$$\int \frac{f'(x)}{\cos^2[f(x)]} dx = \text{tg}[f(x)] + C$$

$$\int \frac{f'(x)}{\text{sen}^2[f(x)]} dx = -\text{cot g}[f(x)] + C$$

$$\int \frac{f'(x)}{\operatorname{ch}^2[f(x)]} dx = \operatorname{th}[f(x)] + C$$

$$\int \frac{f'(x)}{\operatorname{sh}^2[f(x)]} dx = -\operatorname{coth}[f(x)] + C$$

$$\int \frac{f'(x)}{1+f^2(x)} dx = \operatorname{arctg}[f(x)] + C$$

$$\int \frac{f'(x)}{1-f^2(x)} dx = \operatorname{arg th}[f(x)] + C$$

$$\int \frac{f'(x)}{\sqrt{1-f^2(x)}} dx = \operatorname{arc sen}[f(x)] + C$$

$$\int \frac{f'(x)}{\sqrt{1+f^2(x)}} dx = \operatorname{arg sh}[f(x)] + C$$

$$\int \frac{f'(x)}{\sqrt{f^2(x)-1}} dx = \operatorname{arg ch}[f(x)] + C$$

• Ejemplos:

$$\int \frac{1}{\sqrt{1+4x^2}} dx = \int \frac{1}{\sqrt{1+(2x)^2}} dx = \frac{1}{2} \cdot \int \frac{2}{\sqrt{1+(2x)^2}} dx = \frac{1}{2} \cdot \operatorname{arg sh}(2x) + C$$

$$\begin{aligned} \int \frac{1}{3+4x^2} dx &= \int \frac{1}{3 \cdot \left(1 + \frac{4x^2}{3}\right)} dx = \frac{1}{3} \cdot \int \frac{1}{1 + \frac{4x^2}{3}} dx = \frac{1}{3} \cdot \int \frac{1}{1 + \left(\frac{2x}{\sqrt{3}}\right)^2} dx = \frac{1}{3} \cdot \frac{1}{\frac{2}{\sqrt{3}}} \int \frac{\frac{2}{\sqrt{3}}}{1 + \left(\frac{2x}{\sqrt{3}}\right)^2} dx = \\ &= \frac{1}{2 \cdot \sqrt{3}} \cdot \int \frac{\frac{2}{\sqrt{3}}}{1 + \left(\frac{2x}{\sqrt{3}}\right)^2} dx = \frac{1}{2 \cdot \sqrt{3}} \cdot \operatorname{arctg}\left(\frac{2x}{\sqrt{3}}\right) + C \end{aligned}$$

$$\int x \cdot \cos x^2 dx = \frac{1}{2} \cdot \int \cos x^2 \cdot 2x dx = \frac{1}{2} \cdot \operatorname{sen} x^2 + C$$

Integrales racionales básicas:

- $$\int \frac{1}{a^2 + x^2} dx = \int \frac{1}{a^2 \cdot \left(1 + \frac{x^2}{a^2}\right)} dx = \frac{1}{a^2} \cdot \int \frac{1}{1 + \left(\frac{x}{a}\right)^2} dx = \frac{1}{a^2} \cdot a \cdot \int \frac{\frac{1}{a}}{1 + \left(\frac{x}{a}\right)^2} dx = \frac{1}{a} \operatorname{arc\,tg}\left(\frac{x}{a}\right)$$
- $$\int \frac{1}{a^2 - x^2} dx = \int \frac{1}{a^2 \cdot \left(1 - \frac{x^2}{a^2}\right)} dx = \frac{1}{a^2} \cdot \int \frac{1}{1 - \left(\frac{x}{a}\right)^2} dx = \frac{1}{a^2} \cdot a \cdot \int \frac{\frac{1}{a}}{1 - \left(\frac{x}{a}\right)^2} dx = \frac{1}{a} \operatorname{arg\,th}\left(\frac{x}{a}\right)$$
- $$\int \frac{1}{a^2 + (bx)^2} dx = \int \frac{1}{a^2 \cdot \left(1 + \frac{(bx)^2}{a^2}\right)} dx = \frac{1}{a^2} \cdot \int \frac{1}{1 + \left(\frac{bx}{a}\right)^2} dx = \frac{1}{a^2} \cdot \frac{a}{b} \cdot \int \frac{\frac{b}{a}}{1 + \left(\frac{bx}{a}\right)^2} dx = \frac{1}{ab} \operatorname{arc\,tg}\left(\frac{bx}{a}\right)$$
- $$\int \frac{1}{a^2 - (bx)^2} dx = \int \frac{1}{a^2 \cdot \left(1 - \frac{(bx)^2}{a^2}\right)} dx = \frac{1}{a^2} \cdot \int \frac{1}{1 - \left(\frac{bx}{a}\right)^2} dx = \frac{1}{a^2} \cdot \frac{a}{b} \cdot \int \frac{\frac{b}{a}}{1 - \left(\frac{bx}{a}\right)^2} dx = \frac{1}{ab} \operatorname{arg\,th}\left(\frac{bx}{a}\right)$$
- $$\int \frac{1}{a^2 + (bx + c)^2} dx = \int \frac{1}{a^2 \cdot \left(1 + \frac{(bx + c)^2}{a^2}\right)} dx = \frac{1}{a^2} \cdot \int \frac{1}{1 + \left(\frac{bx + c}{a}\right)^2} dx = \frac{1}{a^2} \cdot \frac{a}{b} \cdot \int \frac{\frac{b}{a}}{1 + \left(\frac{bx + c}{a}\right)^2} dx =$$

$$= \frac{1}{ab} \operatorname{arc\,tg}\left(\frac{bx + c}{a}\right)$$
- $$\int \frac{1}{a^2 - (bx + c)^2} dx = \dots = \frac{1}{ab} \operatorname{arg\,th}\left(\frac{bx + c}{a}\right)$$

Integrales irracionales básicas:

$$(a > 0); \int \frac{1}{\sqrt{a^2 - x^2}} dx = \int \frac{1}{\sqrt{a^2 \left(1 - \frac{x^2}{a^2}\right)}} dx = \frac{1}{a} \cdot \int \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} dx = \frac{1}{a} \cdot a \cdot \int \frac{\frac{1}{a}}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} dx = \operatorname{arc\,sen}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \dots = \operatorname{arg\,sh}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \dots = \operatorname{arg\,ch}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{\sqrt{a^2 - (bx)^2}} dx = \dots = \frac{1}{b} \operatorname{arc\,sen}\left(\frac{bx}{a}\right)$$

$$\int \frac{1}{\sqrt{a^2 + (bx)^2}} dx = \dots = \frac{1}{b} \operatorname{arg\,sh}\left(\frac{bx}{a}\right)$$

$$\int \frac{1}{\sqrt{(bx)^2 - a^2}} dx = \dots = \frac{1}{b} \operatorname{arg\,ch}\left(\frac{bx}{a}\right)$$

$$\int \frac{1}{\sqrt{a^2 - (bx+c)^2}} dx = \dots = \frac{1}{b} \operatorname{arc\,sen}\left(\frac{bx+c}{a}\right)$$

$$\int \frac{1}{\sqrt{a^2 + (bx+c)^2}} dx = \dots = \frac{1}{b} \operatorname{arg\,sh}\left(\frac{bx+c}{a}\right)$$

$$\int \frac{1}{\sqrt{(bx+c)^2 - a^2}} dx = \dots = \frac{1}{b} \operatorname{arg\,ch}\left(\frac{bx+c}{a}\right)$$